

Exam C Spring 2007

FINAL ANSWER KEY

<i>Question #</i>	<i>Answer</i>
1	D
2	E
3	B
4	A
5	E
6	A
7	E
8	B
9	B
10	E
11	D
12	A
13	C
14	E
15	D
16	C
17	A
18	C
19	C
20	D

<i>Question #</i>	<i>Answer</i>
21	A
22	D
23	C
24	E
25	A
26	B
27	D
28	B
29	E
30	C
31	A
32	E
33	D
34	E
35	E
36	B
37	E
38	B
39	C
40	D

****BEGINNING OF EXAMINATION****

- 1.** For a dental policy, you are given:
- (i) Ground-up losses follow an exponential distribution with mean θ .
 - (ii) Losses under 50 are not reported to the insurer.
 - (iii) For each loss over 50, there is a deductible of 50 and a policy limit of 350.
 - (iv) A random sample of five claim payments for this policy is:

50 150 200 350⁺ 350⁺

where + indicates that the original loss exceeds 400.

Determine the likelihood function $L(\theta)$.

(A) $\frac{1}{\theta^5} e^{-\frac{1100}{\theta}}$

(B) $\frac{1}{\theta^5} e^{-\frac{1300}{\theta}}$

(C) $\frac{1}{\theta^5} e^{-\frac{1350}{\theta}}$

(D) $\frac{1}{\theta^3} e^{-\frac{1100}{\theta}}$

(E) $\frac{1}{\theta^3} e^{-\frac{1350}{\theta}}$

2. For a group of risks, you are given:

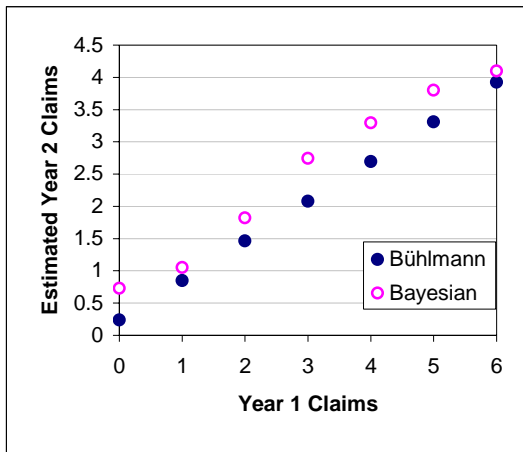
- (i) The number of claims for each risk follows a binomial distribution with parameters $m = 6$ and q .
- (ii) The values of q range from 0.1 to 0.6.

During Year 1, k claims are observed for a randomly selected risk.

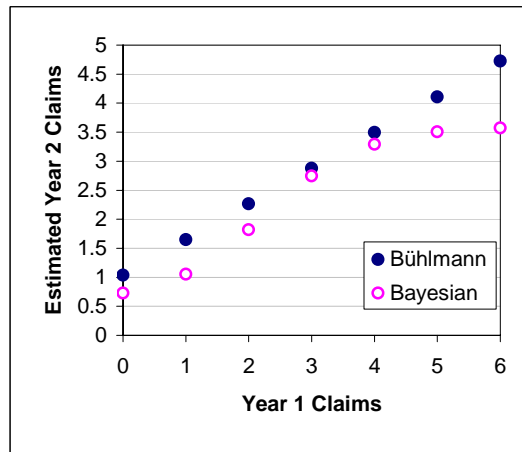
For the same risk, both Bayesian and Bühlmann credibility estimates of the number of claims in Year 2 are calculated for $k = 0, 1, 2, \dots, 6$.

Determine the graph that is consistent with these estimates.

(A)

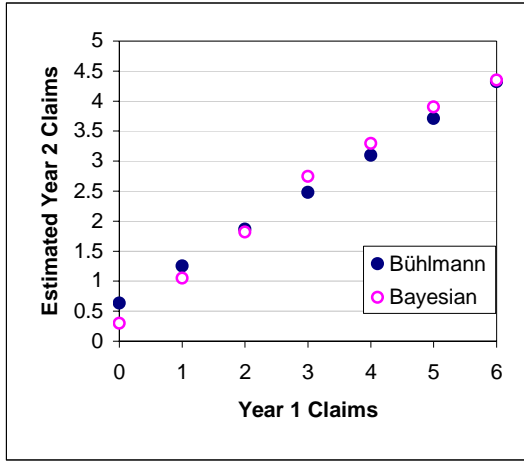


(B)

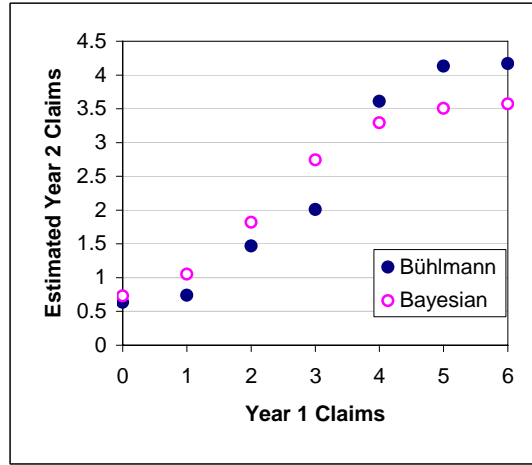


2. (Continued)

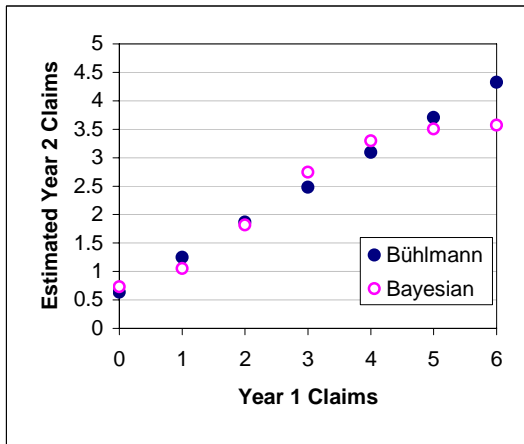
(C)



(D)



(E)



3. You are given:

- (i) Conditional on $Q = q$, the random variables X_1, X_2, \dots, X_m are independent and follow a Bernoulli distribution with parameter q .
- (ii) $S_m = X_1 + X_2 + \dots + X_m$
- (iii) The distribution of Q is beta with $a = 1$, $b = 99$, and $\theta = 1$.

Determine the variance of the marginal distribution of S_{101} .

- (A) 1.00
- (B) 1.99
- (C) 9.09
- (D) 18.18
- (E) 25.25

4. You are given the following information for a stock with current price 0.25:

- (i) The price of the stock is lognormally distributed with continuously compounded expected annual rate of return $\alpha = 0.15$.
- (ii) The dividend yield of the stock is zero.
- (iii) The volatility of the stock is $\sigma = 0.35$.

Using the procedure described in the McDonald text, determine the upper bound of the 90% confidence interval for the price of the stock in 6 months.

- (A) 0.393
- (B) 0.425
- (C) 0.451
- (D) 0.486
- (E) 0.529

5. You are given:

- (i) A computer program simulates $n = 1000$ pseudo- $U(0, 1)$ variates.
- (ii) The variates are grouped into $k = 20$ ranges of equal length.
- (iii) $\sum_{j=1}^{20} O_j^2 = 51,850$
- (iv) The Chi-square goodness-of-fit test for $U(0, 1)$ is performed.

Determine the result of the test.

- (A) Do not reject H_0 at the 0.10 significance level.
- (B) Reject H_0 at the 0.10 significance level, but not at the 0.05 significance level.
- (C) Reject H_0 at the 0.05 significance level, but not at the 0.025 significance level.
- (D) Reject H_0 at the 0.025 significance level, but not at the 0.01 significance level.
- (E) Reject H_0 at the 0.01 significance level.

6. An insurance company sells two types of policies with the following characteristics:

Type of Policy	Proportion of Total Policies	Poisson Annual Claim Frequency
I	θ	$\lambda = 0.50$
II	$1 - \theta$	$\lambda = 1.50$

A randomly selected policyholder is observed to have one claim in Year 1.

For the same policyholder, determine the Bühlmann credibility factor Z for Year 2.

- (A) $\frac{\theta - \theta^2}{1.5 - \theta^2}$
- (B) $\frac{1.5 - \theta}{1.5 - \theta^2}$
- (C) $\frac{2.25 - 2\theta}{1.5 - \theta^2}$
- (D) $\frac{2\theta - \theta^2}{1.5 - \theta^2}$
- (E) $\frac{2.25 - 2\theta^2}{1.5 - \theta^2}$

7. You are given:

(i)

Claim Size	Number of Claims
(0, 50]	30
(50, 100]	36
(100, 200]	18
(200, 400]	16

(ii) Claim sizes within each interval are uniformly distributed.

(iii) The second moment of the uniform distribution on $(a, b]$ is $\frac{b^3 - a^3}{3(b - a)}$.

Estimate $E[(X \wedge 350)^2]$, the second moment of the claim size distribution subject to a limit of 350.

(A) 18,362

(B) 18,950

(C) 20,237

(D) 20,662

(E) 20,750

8. Annual aggregate losses for a dental policy follow the compound Poisson distribution with $\lambda = 3$. The distribution of individual losses is:

Loss	Probability
1	0.4
2	0.3
3	0.2
4	0.1

Calculate the probability that aggregate losses in one year do not exceed 3.

- (A) Less than 0.20
- (B) At least 0.20, but less than 0.40
- (C) At least 0.40, but less than 0.60
- (D) At least 0.60, but less than 0.80
- (E) At least 0.80

9. You are given:

- (i) For a company, the workers compensation lost time claim amounts follow the Pareto distribution with $\alpha = 2.8$ and $\theta = 36$.
- (ii) The cumulative distribution of the frequency of these claims is:

Frequency	Cumulative Probability
0	0.5556
1	0.8025
2	0.9122
3	0.9610
4	0.9827
5	0.9923
\vdots	\vdots

- (iii) Each claim is subject to a deductible of 5 and a maximum payment of 30.

Use the uniform (0, 1) random number 0.981 and the inversion method to generate the simulated number of claims.

Use as many of the following uniform (0, 1) random numbers as necessary, beginning with the first, and the inversion method to generate the claim amounts.

0.571 0.932 0.303 0.471 0.878

Calculate the total of the company's simulated claim payments.

- (A) 37.7
- (B) 41.9
- (C) 56.8
- (D) 64.9
- (E) 84.9

- 10.** A random sample of observations is taken from a shifted exponential distribution with probability density function:

$$f(x) = \frac{1}{\theta} e^{-(x-\delta)/\theta}, \quad \delta < x < \infty$$

The sample mean and median are 300 and 240, respectively.

Estimate δ by matching these two sample quantities to the corresponding population quantities.

- (A) Less than 40
- (B) At least 40, but less than 60
- (C) At least 60, but less than 80
- (D) At least 80, but less than 100
- (E) At least 100

11. Three policyholders have the following claims experience over three months:

Policyholder	Month 1	Month 2	Month 3	Mean	Variance
I	4	6	5	5	1
II	8	11	8	9	3
III	5	7	6	6	1

Nonparametric empirical Bayes estimation is used to estimate the credibility premium in Month 4.

Calculate the credibility factor Z .

- (A) 0.57
- (B) 0.68
- (C) 0.80
- (D) 0.87
- (E) 0.95

12. For 200 auto accident claims you are given:

- (i) Claims are submitted t months after the accident occurs, $t = 0, 1, 2, \dots$
- (ii) There are no censored observations.
- (iii) $\hat{S}(t)$ is calculated using the Kaplan-Meier product limit estimator.
- (iv) $c_S^2(t) = \frac{\widehat{Var}[\hat{S}(t)]}{\hat{S}(t)^2}$, where $\widehat{Var}[\hat{S}(t)]$ is calculated using Greenwood's approximation.
- (v) $\hat{S}(8) = 0.22$, $\hat{S}(9) = 0.16$, $c_S^2(9) = 0.02625$, $c_S^2(10) = 0.04045$

Determine the number of claims that were submitted to the company 10 months after an accident occurred.

- (A) 10
- (B) 12
- (C) 15
- (D) 17
- (E) 18

- 13.** The loss severity random variable X follows the exponential distribution with mean 10,000.

Determine the coefficient of variation of the excess loss variable $Y = \max(X - 30000, 0)$.

- (A) 1.0
- (B) 3.0
- (C) 6.3
- (D) 9.0
- (E) 39.2

14. You are given:

- (i) Twenty claim amounts are randomly selected from a Pareto distribution with $\alpha = 2$ and unknown θ .
- (ii) The maximum likelihood estimate of θ is 7.0.
- (iii) $\sum \ln(x_i + 7.0) = 49.01$
- (iv) $\sum \ln(x_i + 3.1) = 39.30$

You use the likelihood ratio test to test the hypothesis that $\theta = 3.1$.

Determine the result of the test.

- (A) Do not reject H_0 at the 0.10 significance level.
- (B) Reject H_0 at the 0.10 significance level, but not at the 0.05 significance level.
- (C) Reject H_0 at the 0.05 significance level, but not at the 0.025 significance level.
- (D) Reject H_0 at the 0.025 significance level, but not at the 0.01 significance level.
- (E) Reject H_0 at the 0.01 significance level.

15. You are given:

- (i) The number of claims for each policyholder has a binomial distribution with parameters $m = 8$ and q .
- (ii) The prior distribution of q is beta with parameters a (unknown), $b = 9$, and $\theta = 1$.
- (iii) A randomly selected policyholder had the following claims experience:

Year	Number of Claims
1	2
2	k

- (iv) The Bayesian credibility estimate for the expected number of claims in Year 2 based on the Year 1 experience is 2.54545.
- (v) The Bayesian credibility estimate for the expected number of claims in Year 3 based on the Year 1 and Year 2 experience is 3.73333.

Determine k .

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 8

- 16.** You use a uniform kernel density estimator with $b = 50$ to smooth the following workers compensation loss payments:

82 126 161 294 384

If $\hat{F}(x)$ denotes the estimated distribution function and $F_5(x)$ denotes the empirical distribution function, determine $|\hat{F}(150) - F_5(150)|$.

- (A) Less than 0.011
- (B) At least 0.011, but less than 0.022
- (C) At least 0.022, but less than 0.033
- (D) At least 0.033, but less than 0.044
- (E) At least 0.044

17. You are given:

- (i) Aggregate losses follow a compound model.
- (ii) The claim count random variable has mean 100 and standard deviation 25.
- (iii) The single-loss random variable has mean 20,000 and standard deviation 5000.

Determine the normal approximation to the probability that aggregate claims exceed 150% of expected costs.

- (A) 0.023
- (B) 0.056
- (C) 0.079
- (D) 0.092
- (E) 0.159

18. You are given:

- (i) The distribution of the number of claims per policy during a one-year period for a block of 3000 insurance policies is:

Number of Claims per Policy	Number of Policies
0	1000
1	1200
2	600
3	200
4+	0

- (ii) You fit a Poisson model to the number of claims per policy using the method of maximum likelihood.
- (iii) You construct the large-sample 90% confidence interval for the mean of the underlying Poisson model that is symmetric around the mean.

Determine the lower end-point of the confidence interval.

- (A) 0.95
- (B) 0.96
- (C) 0.97
- (D) 0.98
- (E) 0.99

19. The price of a non dividend-paying stock is to be estimated using simulation. It is known that:

(i) The price S_t follows the lognormal distribution: $\ln\left(\frac{S_t}{S_0}\right) \sim N\left[\left(\alpha - \frac{\sigma^2}{2}\right)t, \sigma^2 t\right]$

(ii) $S_0 = 50$, $\alpha = 0.15$, and $\sigma = 0.30$.

Using the following uniform (0, 1) random numbers and the inversion method, three prices for two years from the current date are simulated.

0.9830 0.0384 0.7794

Calculate the mean of the three simulated prices.

- (A) Less than 75
- (B) At least 75, but less than 85
- (C) At least 85, but less than 95
- (D) At least 95, but less than 115
- (E) At least 115

- 20.** You use the Kolmogorov-Smirnov goodness-of-fit test to assess the fit of the natural logarithms of $n = 200$ losses to a distribution with distribution function F^* .

You are given:

- (i) The largest value of $|F_n(x) - F^*(x)|$ occurs for some x between 4.26 and 4.42.

(ii)

Observed x	$F^*(x)$	$F_n(x-)$	$F_n(x)$
4.26	0.584	0.505	0.510
4.30	0.599	0.510	0.515
4.35	0.613	0.515	0.520
4.36	0.621	0.520	0.525
4.39	0.636	0.525	0.530
4.42	0.638	0.530	0.535

- (iii) Commonly used large-sample critical values for this test are $1.22/\sqrt{n}$ for $\alpha = 0.10$, $1.36/\sqrt{n}$ for $\alpha = 0.05$, $1.52/\sqrt{n}$ for $\alpha = 0.02$ and $1.63/\sqrt{n}$ for $\alpha = 0.01$.

Determine the result of the test.

- (A) Do not reject H_0 at the 0.10 significance level.
- (B) Reject H_0 at the 0.10 significance level, but not at the 0.05 significance level.
- (C) Reject H_0 at the 0.05 significance level, but not at the 0.02 significance level.
- (D) Reject H_0 at the 0.02 significance level, but not at the 0.01 significance level.
- (E) Reject H_0 at the 0.01 significance level.

21. You are given:

- (i) Losses in a given year follow a gamma distribution with parameters α and θ , where θ does not vary by policyholder.
- (ii) The prior distribution of α has mean 50.
- (iii) The Bühlmann credibility factor based on two years of experience is 0.25.

Calculate $\text{Var}(\alpha)$.

- (A) Less than 10
- (B) At least 10, but less than 15
- (C) At least 15, but less than 20
- (D) At least 20, but less than 25
- (E) At least 25

22. You are given:

- (i) A Cox proportional hazards model was used to study losses on two groups of policies.
- (ii) A single covariate z was used with $z = 0$ for a policy in Group 1 and $z = 1$ for a policy in Group 2.
- (iii) A sample of three policies was taken from each group. The losses were:

Group 1:	275	325	520
Group 2:	215	250	300

- (iv) The baseline survival function is $S_0(x) = \left(\frac{200}{x}\right)^\alpha$, $x > 200$, $\alpha > 0$.

Calculate the maximum likelihood estimate of the coefficient β .

- (A) -0.92
- (B) -0.40
- (C) 0.40
- (D) 0.92
- (E) 2.51

23. For an insurance company you are given:

- (i) The initial surplus is $u = 1$.
- (ii) An annual premium of 2 is collected at the beginning of each year.
- (iii) The distribution of annual losses is:

Loss	Probability
0	0.6
2	0.3
6	0.1

- (iv) Investment income is earned on the available capital at the beginning of the year at an annual rate of 10 percent.
- (v) Losses are paid and investment income is collected at the end of the year.
- (vi) There are no other cash flows.

Calculate the discrete time finite horizon ruin probability $\tilde{\psi}(1,3)$.

- (A) 0.100
- (B) 0.190
- (C) 0.199
- (D) 0.217
- (E) 0.235

24. For a portfolio of policies, you are given:

(i) Losses follow a Weibull distribution with parameters θ and τ .

(ii) A sample of 16 losses is :

54 70 75 81 84 88 97 105 109 114 122 125 128 139 146 153

(iii) The parameters are to be estimated by percentile matching using the 20th and 70th smoothed empirical percentiles.

Calculate the estimate of θ .

(A) Less than 100

(B) At least 100, but less than 105

(C) At least 105, but less than 110

(D) At least 110, but less than 115

(E) At least 115

25. You are given:

- (i) During a single 5-year period, 100 policies had the following total claims experience:

Number of Claims in Year 1 through Year 5	Number of Policies
0	46
1	34
2	13
3	5
4	2

- (ii) The number of claims per year follows a Poisson distribution.
(iii) Each policyholder was insured for the entire period.

A randomly selected policyholder had 3 claims over the period.

Using semiparametric empirical Bayes estimation, determine the Bühlmann estimate for the number of claims in Year 6 for the same policyholder.

- (A) Less than 0.25
(B) At least 0.25, but less than 0.50
(C) At least 0.50, but less than 0.75
(D) At least 0.75, but less than 1.00
(E) At least 1.00

26. The following table was calculated based on loss amounts for a group of motorcycle insurance policies:

c_j	d_j	u_j	x_j	P_j
250	6	0	1	0
500	6	0	2	5
1000	7	1	4	9
2750	0	1	7	11
5500	0	1	1	3
6000	0	0	1	1
10,000	0	0	0	0

You are given $\alpha = 1$ and $\beta = 0$.

Using the procedure in the *Loss Models* text, estimate the probability that a policy with a deductible of 500 will have a claim payment in excess of 5500.

- (A) Less than 0.13
- (B) At least 0.13, but less than 0.16
- (C) At least 0.16, but less than 0.19
- (D) At least 0.19, but less than 0.22
- (E) At least 0.22

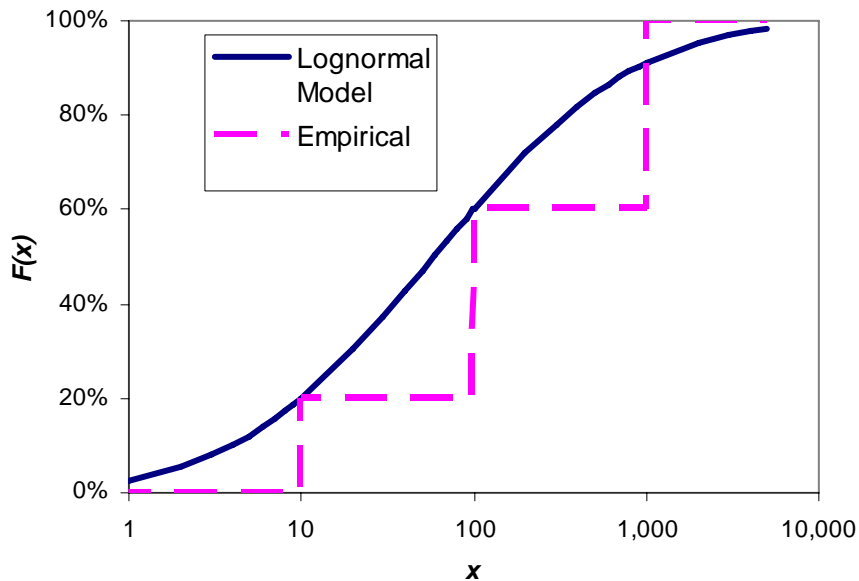
27. You are given the distortion function:

$$g(x) = \sqrt{x}$$

Calculate the distortion risk measure for losses that follow the Pareto distribution with $\theta = 1000$ and $\alpha = 4$.

- (A) Less than 300
- (B) At least 300, but less than 600
- (C) At least 600, but less than 900
- (D) At least 900, but less than 1200
- (E) At least 1200

28. You are given the following graph of cumulative distribution functions:



Determine the difference between the mean of the lognormal model and the mean of the data.

- (A) Less than 50
- (B) At least 50, but less than 150
- (C) At least 150, but less than 350
- (D) At least 350, but less than 750
- (E) At least 750

29. For a policy that covers both fire and wind losses, you are given:

- (i) A sample of fire losses was 3 and 4.
- (ii) Wind losses for the same period were 0 and 3.
- (iii) Fire and wind losses are independent, but do not have identical distributions.

Based on the sample, you estimate that adding a policy deductible of 2 per wind claim will eliminate 20% of the insured loss.

Determine the bootstrap approximation to the mean square error of the estimate.

- (A) Less than 0.006
- (B) At least 0.006, but less than 0.008
- (C) At least 0.008, but less than 0.010
- (D) At least 0.010, but less than 0.012
- (E) At least 0.012

30. You are given:

- (i) Conditionally, given β , an individual loss X follows the exponential distribution with probability density function:

$$f(x|\beta) = \frac{1}{\beta} \exp(-x/\beta), \quad 0 < x < \infty.$$

- (ii) The prior distribution of β is inverse gamma with probability density function:

$$\pi(\beta) = \frac{c^2}{\beta^3} \exp(-c/\beta), \quad 0 < \beta < \infty.$$

- (iii) $\int_0^\infty \frac{1}{y^n} \exp(-a/y) dy = \frac{(n-2)!}{a^{n-1}}, \quad n = 2, 3, 4, \dots$

Given that the observed loss is x , calculate the mean of the posterior distribution of β .

- (A) $\frac{1}{x+c}$
- (B) $\frac{2}{x+c}$
- (C) $\frac{x+c}{2}$
- (D) $x+c$
- (E) $2(x+c)$

31. You are given:

- (i) An insurance company records the following ground-up loss amounts, which are generated by a policy with a deductible of 100:

120 180 200 270 300 1000 2500

- (ii) Losses less than 100 are not reported to the company.
(iii) Losses are modeled using a Pareto distribution with parameters $\theta = 400$ and α .

Use the maximum likelihood estimate of α to estimate the expected loss with no deductible.

- (A) Less than 500
(B) At least 500, but less than 1000
(C) At least 1000, but less than 1500
(D) At least 1500, but less than 2000
(E) At least 2000

- 32.** You are given n years of claim data originating from a large number of policies. You are asked to use the Bühlmann-Straub credibility model to estimate the expected number of claims in year $n + 1$.

Which of conditions (A), (B), or (C) are required by the model?

- (A) All policies must have an equal number of exposure units.
- (B) Each policy must have a Poisson claim distribution.
- (C) There must be at least 1082 exposure units.
- (D) Each of (A), (B), and (C) is required.
- (E) None of (A), (B), or (C) is required.

33. You are given:

- (i) Eight people join an exercise program on the same day. They stay in the program until they reach their weight loss goal or switch to a diet program.
- (ii) Experience for each of the eight members is shown below:

Member	Time at Which...	
	Reach Weight Loss Goal	Switch to Diet Program
1		4
2		8
3	8	
4	12	
5		12
6	12	
7	22	
8	36	

- (iii) The variable of interest is time to reach weight loss goal.

Using the Nelson-Aalen estimator, calculate the upper limit of the symmetric 90% linear confidence interval for the cumulative hazard rate function $H(12)$.

- (A) 0.85
- (B) 0.92
- (C) 0.95
- (D) 1.06
- (E) 1.24

- 34.** The price of a stock in seven consecutive months is:

Month	Price
1	54
2	56
3	48
4	55
5	60
6	58
7	62

Based on the procedure described in the McDonald text, calculate the annualized expected return of the stock.

- (A) Less than 0.28
- (B) At least 0.28, but less than 0.29
- (C) At least 0.29, but less than 0.30
- (D) At least 0.30, but less than 0.31
- (E) At least 0.31

35. The observation from a single experiment has distribution:

$$\Pr(D = d \mid G = g) = g^{(1-d)}(1-g)^d \quad \text{for } d = 0, 1$$

The prior distribution of G is:

$$\Pr\left(G = \frac{1}{5}\right) = \frac{3}{5} \quad \text{and} \quad \Pr\left(G = \frac{1}{3}\right) = \frac{2}{5}$$

Calculate $\Pr\left(G = \frac{1}{3} \mid D = 0\right)$.

(A) $\frac{2}{19}$

(B) $\frac{3}{19}$

(C) $\frac{1}{3}$

(D) $\frac{9}{19}$

(E) $\frac{10}{19}$

36. For a portfolio of insurance risks, aggregate losses per year per exposure follow a normal distribution with mean θ and standard deviation 1000, with θ varying by class as follows:

Class	θ	Percent of Risks in Class
X	2000	60%
Y	3000	30%
Z	4000	10%

A randomly selected risk has the following experience over three years:

Year	Number of Exposures	Aggregate Losses
1	24	24,000
2	30	36,000
3	26	28,000

Calculate the Bühlmann-Straub estimate of the mean aggregate losses per year per exposure in Year 4 for this risk.

- (A) 1100
- (B) 1138
- (C) 1696
- (D) 2462
- (E) 2500

- 37.** Simulation is used to estimate the value of the cumulative distribution function at 300 of the exponential distribution with mean 100.

Determine the minimum number of simulations so that there is at least a 99% probability that the estimate is within $\pm 1\%$ of the correct value.

- (A) 35
- (B) 100
- (C) 1418
- (D) 2013
- (E) 3478

38. You are given:

- (i) All members of a mortality study are observed from birth. Some leave the study by means other than death.
- (ii) $s_3 = 1$, $s_4 = 3$
- (iii) The following Kaplan-Meier product-limit estimates were obtained:
$$S_n(y_3) = 0.65, \quad S_n(y_4) = 0.50, \quad S_n(y_5) = 0.25$$
- (iv) Between times y_4 and y_5 , six observations were censored.
- (v) Assume no observations were censored at the times of deaths.

Determine s_5 .

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

39. You are given:

- (i) The frequency distribution for the number of losses for a policy with no deductible is negative binomial with $r = 3$ and $\beta = 5$.
- (ii) Loss amounts for this policy follow the Weibull distribution with $\theta = 1000$ and $\tau = 0.3$.

Determine the expected number of payments when a deductible of 200 is applied.

- (A) Less than 5
- (B) At least 5, but less than 7
- (C) At least 7, but less than 9
- (D) At least 9, but less than 11
- (E) At least 11

40. You are given:

Loss Experience	Number of Policies
0 claims	1600
1 or more claims	400

Using the normal approximation, determine the upper bound of the symmetric 95% confidence interval for the probability that a single policy has 1 or more claims.

- (A) 0.200
- (B) 0.208
- (C) 0.215
- (D) 0.218
- (E) 0.223

****END OF EXAMINATION****